

Can Modern Nuclear Hamiltonians Tolerate a Bound Tetraneutron?

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(Dated: June 5, 2005)

I show that it does not seem possible to change modern nuclear Hamiltonians to bind a tetraneutron without destroying many other successful predictions of those Hamiltonians. This means that, should a recent experimental claim of a bound tetraneutron be confirmed, our understanding of nuclear forces will have to be significantly changed. I also point out some errors in previous theoretical studies of this problem.

PACS numbers: 13.75.Cs, 21.30.-x, 21.45.+v, 21.60.Ka, 21.65.+f

An experimental claim of the existence of a bound tetraneutron cluster (${}^4\text{n}$) was made last year [1]. Since then a number of theoretical attempts to obtain such bound systems have been made, with the conclusion that nuclear potentials do not bind four neutrons [2, 3, 4]. However these studies have been made with simplified Hamiltonians and only approximate solutions of the four-neutron problem. In this paper I use modern realistic nuclear Hamiltonians that provide a good description of nuclei up to $A = 10$ and accurate Green's function Monte Carlo (GFMC) calculations to improve this situation. (A list of earlier experimental and theoretical studies, also with generally negative conclusions, may be found in [2, 3, 4].)

A series of papers [5, 6, 7], have presented the development of GFMC for calculations of light nuclei (so far up to $A = 10$) using realistic two-nucleon (NN) and three-nucleon (NNN) potentials. For a given Hamiltonian, the method obtains ground and low-lying excited state energies with an accuracy of 1–2%. I use this method in the present study; tests similar to those reported in the above papers have verified that the energies reported here have similar accuracies, with two exceptions: 1) when the energies are very close to 0, the error is probably a few 100 keV; and 2) the ${}^4\text{H}$ calculations contain a technical difficulty that might be introducing systematic errors of up to 1 MeV. The reader is referred to [8] for a review of the nuclear GFMC method or to the previously cited papers for complete details of how the present calculations were made.

By using the Argonne v_{18} NN potential (AV18) [9] and including two- and three-pion exchange NNN potentials, a series of model Hamiltonians (the Illinois models) were constructed that reproduce energies for $A = 3$ –10 nuclei with rms errors of 0.6–1.0 MeV [10]. The best model, the AV18+Illinois-2 (AV18/IL2) model, is used in the present study.

GFMC starts with a trial wave function, Ψ_T , which determines the quantum numbers of the state being computed. For p -shell nuclei studied in the above references, the Jastrow part of Ψ_T contains four nucleons with an alpha-particle wave function and $A - 4$ nucleons in p -shell

orbitals. This is multiplied by a product of non-central two- and three-particle correlation operators. I use Ψ_T for ${}^4\text{n}$ with the same structure except there are two neutrons in a 1S_0 configuration and two in the p -shell. The total J^π of the ${}^4\text{n}$ ground state is assumed to be 0^+ . There are two possible symmetry states in the p -shell using LS coupling: ${}^1S[22]$ and ${}^3P[211]$; both are used in these calculations. I could find no Ψ_T that gave a negative energy for ${}^4\text{n}$ using the AV18/IL2 model. GFMC calculations, using propagation to very large imaginary time ($\tau = 1.6 \text{ MeV}^{-1}$), also produced positive energies that steadily decreased as the rms radius of the system increased.

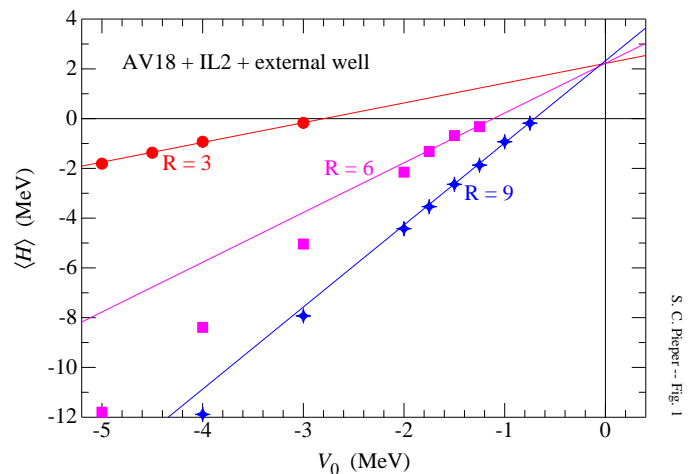


FIG. 1: Energies of ${}^4\text{n}$ in external wells versus the well-depth parameter, V_0 .

In a second study, I added artificial external wells of Woods-Saxon shape to the AV18/IL2 Hamiltonian and used GFMC to find the resulting total energies of the four neutrons. Figure 1 shows results for wells with radii $R = 3, 6$, and 9 fm (all have diffuseness parameters of 0.65 fm) and varying depth parameter, V_0 . It seems clear that four neutrons become unbound (have positive energy) significantly before the well depth is reduced to zero. Linear fits to the least bound energies for each

Woods Saxon radius parameter are also shown; these extrapolate to an energy of +2 MeV when the external well is removed. (These least-bound solutions have large rms radii. A transition from the indicated linear behavior to a steeper linear behavior is observed for deeper wells; this transition is associated with a change to much smaller rms radii solutions. The steeper fits of course extrapolate to much larger positive energies.) This suggests that there might be a ^4n resonance near 2 MeV, but since the GFMC calculation with no external well shows no indication of stabilizing at that energy, the resonance, if it exists at all, must be very broad. In any case, the AV18/IL2 model does not produce a bound ^4n .

The authors of [1] suggest that only small modifications of existing nuclear Hamiltonians may be necessary to bind four neutrons. To study this possibility, I made a number of modifications to the AV18/IL2 model. In each case the modification was adjusted to bind ^4n with an energy of approximately -0.5 MeV; the consequences of this modification for other nuclei were then computed. Four of the modifications are reported here: long- and moderate-range changes of the NN potential in the 1S_0 partial wave; introduction of an additional NNN potential that acts only in total isospin $T = \frac{3}{2}$ triples; and introduction of a $NNNN$ potential that acts only in $T = 2$ quadruples. In all cases the complete AV18/IL2 Hamiltonian was used with the additional term.

The strong-interaction part of the AV18 NN potential consists of one-pion exchange with the generally accepted value of $f_\pi^2/4\pi = 0.075$, moderate-range terms that are associated with two-pion exchange but which have phenomenologically adjusted strengths, and a short-ranged completely phenomenological part. The potential is written in terms of operators which can be used to produce the potential for any partial wave. By making correlated changes to the radial parts of the central, $\sigma_i \cdot \sigma_j$, $\tau_i \cdot \tau_j$, and $\sigma_i \cdot \sigma_j \tau_i \cdot \tau_j$ operators, one can change the potential in just the $S = 0, T = 1$ partial waves: $^1S_0, ^1D_2, ^1G_4$, etc. A corresponding change to the four \mathbf{L}^2 operator terms can cancel the change in the 1D_2 partial wave, leaving a change to basically only the 1S_0 partial wave.

In the first such modification of the AV18, I changed just the two-pion range part of the 1S_0 partial wave, so as to leave the theoretically well established one-pion part unaffected. Increasing this two-pion strength by 4.9% results in a ^4n energy of $-0.87(3)$ MeV. (The statistical errors in Monte Carlo computed numbers are shown in parentheses only when they exceed unity in the last quoted digit.) As is shown by the points labeled “mod- $^1S_0-2\pi$ ” in Fig. 2, this changes the 1S_0 phase shifts by 12° over a large energy range and produces a bound dineutron (the energy is -0.88 MeV). These changes far exceed those allowed by modern phase shift analysis. A somewhat smaller change that produces a bound ^4n can be made by using the AV1’ potential [11] in the 1S_0 partial wave (and AV18 in the other partial waves); this results

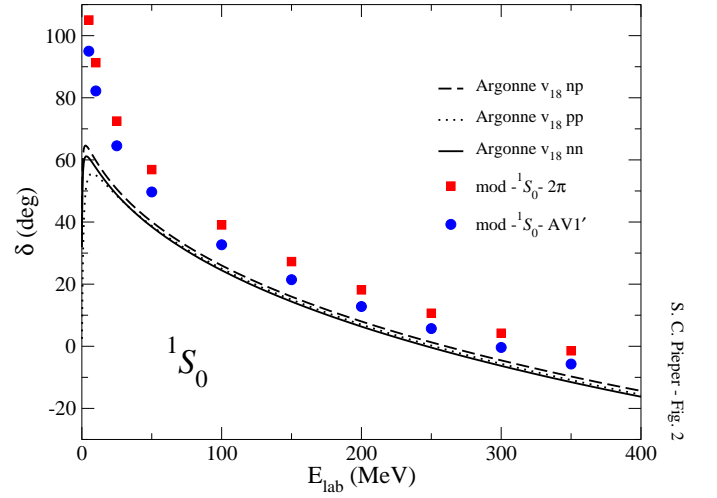


FIG. 2: 1S_0 phase shifts from AV18 and modifications to it. The lines show the pp, pn, and nn phase shifts for the unmodified AV18 while the symbols show the modified results.

in a ^4n energy of -0.52 MeV and about a 50% smaller change in the 1S_0 phase shifts (the points labeled “mod- $^1S_0\text{-AV1'}$ ” in the figure). However again ^2n is bound, this time with an energy of -0.42 MeV. Note that the one-pion-range part of the potential is also changed in mod- $^1S_0\text{-AV1'}$. These ^2n and ^4n bound states are quite diffuse; the rms radii are respectively 2.8 and 3.6 fm for the two ^2n cases and 7.3 and 10.3 fm for the ^4n . The ^4n pair distributions have a peak containing about two pairs with a structure close to that of the ^2n pair distribution and a long tail. Thus the ^4n looks like two widely separated dineutrons.

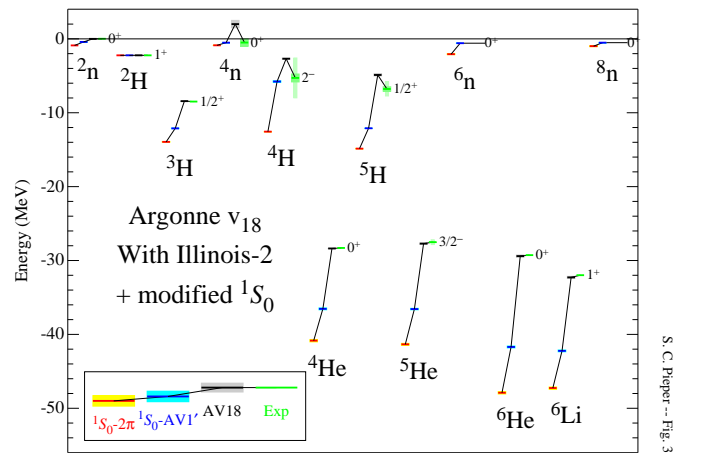


FIG. 3: Energies of nuclei and neutron clusters computed with the AV18/IL2 Hamiltonian with modified NN potentials ($^1S_0-2\pi$ and $^1S_0\text{-AV1'}$) and with no modification (AV18), compared with experimental values for known nuclei.

As noted, these modifications of the 1S_0 potential to produce minimally bound tetraneutrons also produce dineutrons with about the same binding energies; thus

they are physically unacceptable modifications. Figure 3 shows that they also introduce large changes to the binding energies of other nuclei; for example ${}^3\text{H}$ is $\sim 50\%$ overbound and ${}^5\text{H}$ is stable or almost stable against breakup into ${}^3\text{H}+n+n$ as opposed to being a resonance in that channel [12]. Also six and eight neutrons form bound systems, although three and five do not..

The authors of Refs. [2, 3, 4] concluded that the non-realistic Volkov potentials [13] do not bind ${}^4\text{n}$. However these potentials do have bound dineutrons and the above results suggest that they thus might bind ${}^4\text{n}$. To study this I made calculations using the first four Volkov potentials in all partial waves and no NNV potential. These potentials indeed do bind ${}^4\text{n}$ with energies of -0.91 , -1.04 , -0.47 and -0.71 MeV while the ${}^2\text{n}$ energies are -0.56 , -0.60 , -0.35 , and -0.42 MeV, respectively. The rms radii of the ${}^4\text{n}$ systems are all about 11.5 fm, which may explain why these bound states were not discovered in Refs. [2, 3, 4]. The variational energies for ${}^4\text{n}$ with modifications to the AV18/IL2 Hamiltonian are positive; that is, only with GFMC improvement does the system become bound. However for the simpler Volkov potentials, the Ψ_T already give negative energies and the GFMC just improves these energies.

It must be emphasized that these bound ${}^4\text{n}$ results do not at all support an experimentally bound ${}^4\text{n}$. The more than 35-year old Volkov potentials are not realistic; they produce bound ${}^2\text{n}$, with the same binding energies as their deuterons; they have no tensor or $L\cdot S$ terms; and they can not reproduce modern phase shift analyzes in any partial wave. The one thing in their favor is that, by having a space-exchange component, they introduce some saturation in p -shell nuclear binding energies; however with just one radial form they are even simpler than the space-exchange AVX' introduced in Ref [11].

The above results show that it is not possible to bind ${}^4\text{n}$ by modifying the 1S_0 potential without severely disrupting other nuclear properties. The next NN possibility is the 3P_J channel. The net effect of these is a small repulsion in neutron systems. Setting this term to zero had very little effect on ${}^4\text{n}$; one would have to introduce significant attraction to bind ${}^4\text{n}$ and then again many other nuclear properties would be unrealistically changed.

It has been suggested that modifications to the NNV or $NNNV$ potentials, which are experimentally much less constrained than the NN potential, could be used to bind ${}^4\text{n}$. Timofeyuk added a central $NNNV$ potential to bind ${}^4\text{n}$, but found that it resulted in ${}^4\text{He}$ being bound by about 100 MeV [2, 4]. However, as she suggests, one should try less disruptive things. A NNV potential that acts only in $T = \frac{3}{2}$ triples would have the same effect on ${}^4\text{n}$ as one with no isospin dependence, but no effect on ${}^3\text{H}$ and ${}^4\text{He}$ because they contain only $T = \frac{1}{2}$ triples. A $NNNV$ $T = 2$ potential would also not effect ${}^5\text{He}$ and ${}^6\text{Li}$.

To study such possibilities, I added potentials of the

forms

$$V_{ijk}(T=\frac{3}{2}) = V_3 \sum_{cyclic} [Y(r_{ij})Y(r_{jk})]P(T=\frac{3}{2}),$$

$$V_{ijkl}(T=2) = V_4 \sum_{cyclic} [Y(r_{ij})Y(r_{jk})Y(r_{kl})]P(T=2),$$

$$Y(r) = \frac{e^{-m_\pi r}}{m_\pi r} [1 - e^{-(m_\pi r)^2}]^2,$$

to the AV18/IL2 Hamiltonian. Here m_π is the pion mass, the P are projectors onto the indicated isospin states, and V_3 and V_4 were chosen to produce ${}^4\text{n}$ with ~ -0.5 MeV energy. These forms have the longest range that is possible from strong interactions; the cutoff makes the radial forms peak at 1.55 fm. Using more confined radial forms only increases the problems reported below.

It turns out that the couplings must be quite large to produce the minimally bound ${}^4\text{n}$: $V_3 = -440$ and $V_4 = -4750$ MeV, which result in ${}^4\text{n}$ energies of $-0.60(5)$ and $-0.55(6)$ MeV. This can be understood as follows. If the NN potential is used to bind ${}^4\text{n}$, the pairs can sequentially come close enough to feel the attraction; this allows the four neutrons to be in a diffuse, large radius, distribution. However a NNN potential requires three neutrons to simultaneously be relatively close and thus the density of the system must be much higher. Indeed the rms radii of the ${}^4\text{n}$ for the $V_{ijk}(T=\frac{3}{2})$ case is only 1.88 fm, while that for $V_{ijkl}(T=2)$ is 1.61 fm. Such small radii result in kinetic energies that are an order of magnitude more than those for the ${}^4\text{n}$ systems bound by modified 1S_0 potentials; for the $V_{ijk}(T=\frac{3}{2})$ case the expectation value of the kinetic energy is ~ 87 MeV, while those of the NN and NNN potentials are -49 and -38 MeV, respectively. (As is discussed in Ref. [5], GFMC directly computes only $\langle H \rangle$; other expectation values involve extrapolations. Here I have reported the extrapolated potential values and subtracted these from $\langle H \rangle$ to get the kinetic energy.)

The very large coupling constants for the $V_{ijk}(T=\frac{3}{2})$ and $V_{ijkl}(T=2)$ potentials mean that they have a large, even catastrophic, effect on any nuclear system in which they can act. This is shown in Fig. 4; for example $V_{ijk}(T=\frac{3}{2})$ doubles the binding energy of ${}^6\text{Li}$ and triples that of ${}^6\text{He}$, while $V_{ijkl}(T=2)$, which can have no effect on ${}^6\text{Li}$, quadruples the binding energy of ${}^6\text{He}$. As noted before both of these potentials have no effect on ${}^4\text{He}$. Both potentials make ${}^5\text{H}$ stable by more than 25 MeV against ${}^3\text{H}+n+n$. However the most dramatic result of these potentials is that every investigated pure neutron system with $A > 4$ is extremely bound and in fact is the most stable “nucleus” of that A . For $V_{ijk}(T=\frac{3}{2})$ the energies are -62 , -220 , and -650 MeV respectively for $A = 5, 6, 8$, while for $V_{ijkl}(T=2)$ they are -358 , -1370 , and -6690 MeV.

These enormous bindings indicate that matter will collapse with such potentials. This is to be expected for purely attractive many-nucleon potentials. One should

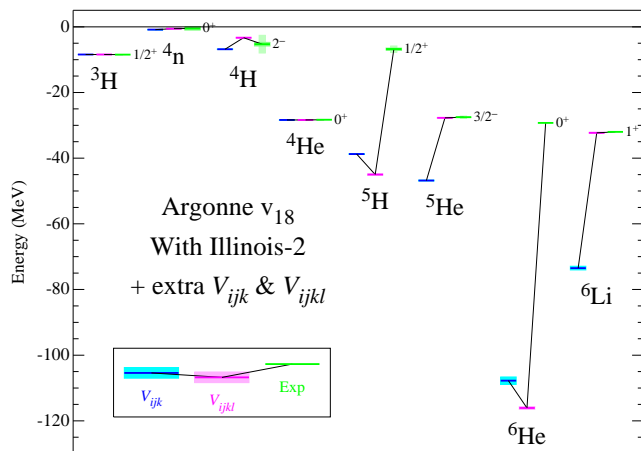


FIG. 4: Energies of nuclei and neutron clusters computed with modified NNN and $NNNN$ potentials.

add a shorter-ranged stronger repulsion to obtain saturation. Such a repulsion might improve the results for $A \geq 6$ nuclei. I attempted to study this by using a repulsive term with Yukawa radial forms of range $2m_\pi$. However, in order to get any appreciable effect on ${}^6\text{He}$, the repulsive coupling has to be made quite large; this then requires at least a doubling of the attraction to still bind ${}^4\text{n}$; this results in potentials that are so strong that the GFMC starts to become unreliable. The apparent impossibility of correcting the $A = 6$ results by such a term may also be seen from the rms radii of the ${}^4\text{n}$ reported above; they are smaller than the experimental value for ${}^6\text{Li}$ and reasonable ${}^6\text{He}$ radii. Thus a short-ranged repulsion that still leaves the ${}^4\text{n}$ bound will certainly result in $A = 6$ nuclei with too small rms radii.

In all of these cases I have made isospin-conserving modifications to the AV18/IL2 Hamiltonian; thus there have been $T = 1$ additions to the NN potential, or a $T = \frac{3}{2}$ addition to the NNN potential, or a $T = 2$ addition to the $NNNN$ potential. It might be objected that the modifications should have been made only for nn pairs or nnn triples or $nnnn$ quadruples since the nuclear force is least well determined for such systems. Such changes would mean much larger charge-symmetry breaking and charge-independence breaking potentials than are presently accepted. But even so, the changes to the NN force, if limited to just nn pairs, would still bind two neutrons which would change the experimental scattering length from ~ -18 fm to a positive value.

Such a nn potential would still bind ${}^6\text{n}$ and ${}^8\text{n}$. I estimate that it would still increase the binding of ${}^3\text{H}$ by 3 MeV while it would have no effect on ${}^3\text{He}$. Thus the Nolen-Schiffer energy for the $A = 3$ system would be some five times too large. Many of the devastating effects shown in Fig. 4 would similarly persist even if the potentials were limited to nnn triples or $nnnn$ quadruples.

In conclusion, should the results of Ref. [1] be confirmed, our current very successful understanding of nuclear forces would have to be severely modified in ways that, at least to me, are not at all obvious.

I thank R. B. Wiringa for many useful suggestions during the course of this work, especially concerning modifications to the AV18 potential; I also thank V. R. Pandharipande for a critical reading of an early version of the manuscript. The calculations were made possible by generous grants of time on the Chiba City Linux cluster of the Mathematics and Computer Science Division, and friendly-user access to the Jazz Linux cluster of the Laboratory Computing Resource Center, both at Argonne National Laboratory. This work is supported by the U. S. Department of Energy, Nuclear Physics Division, under contract No. W-31-109-ENG-38.

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